Reproducing the Pioneer 10 Anomaly with a Cosmic Lens

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May 22, 2007

Abstract

Anderson *et al*. wrote: "We have reported an anomalous, constant acceleration of Pioneer 10 and 11, a = (8.74 ±1.33)·10^{-8} cm/s^2, directed towards the Sun, at distances ~20—70 AU." This anomaly can be reproduced by a concentric multi-layer, onion-like, spherical cosmic lens centered on the Sun with an inner radius of 20 AU. If the effect stops at around 70 AU then the lens has eight or nine layers, and the outer radius of the lens is between 63.55 and 74.08 AU, and the effective refraction index through the eight layers is 1.00055046 and through nine layers is 1.0006858. If the effect extends beyond 74 AU, then there are more than nine layers. The ratios between the volumes of adjacent lens layers is a constant that depends on the error bars for the anomaly. For the given error bars of ±1.33·10^{-8} cm/s^2 that constant adjacent volume ratio is 1.584225. The anomalous acceleration towards the Sun is just the calculated acceleration minus the Newtonian acceleration. The Newtonian acceleration is simply a = GM/r^2, where GM is the heliocentric gravitation constant, and r is the radial distance from the Sun to the spacecraft. The calculated acceleration is just the Newtonian acceleration times the effective index of refraction at the spacecraft's location.

1. The Pioneer 10 Anomaly

John D. Anderson of the Jet Propulsion Laboratory, *et al*.[1][1] wrote: "We have reported an anomalous, constant acceleration of Pioneer 10 and 11, a = (8.74 ±1.33)·10^{-8} cm/s^2, directed towards the Sun, at distances ~20—70 AU." In the MKS units adopted in this paper, that becomes a = (8.74 ±1.33)·10^{-10} m/s^2.

This information completely defines an effect that this paper emulates with a multi-layer, onion-like, concentric spherical shell cosmic lens centered on the Sun. A cosmic lens is defined as a spherical volume of space centered on a gravitating body in which the speed of light is a slower constant inside the volume than outside it. In the case of a multi-layer, concentric spherical shell lens, the speed of light within each spherical shell is slower than it is in the shell (or space) above it and faster than it is in the shell (or space) below it. The ratio of the speed of light within a given shell to the speed of light inside all the shells, e.g., at the Earth, is that shell's effective refraction index. A photon emitted by a spacecraft outside the entire onion would experience successive speed reductions as it crossed successive shell boundaries moving inward towards the Sun.

2. Pioneer 10's Orbit

Any theorist who attempts to explain the Pioneer 10 anomaly needs to know how far away from the Sun the spacecraft was at any moment of time. For that we need some orbital elements for the spacecraft. We don't need the argument of the perihelion or the longitude of the ascending node or the inclination. We only need the semi-major axis, the eccentricity, and the time of the perihelion.

Table I. Pioneer 10 Orbit Observations

Date	R AU	Observation	Hyperbolic Mean anomaly radians	Hyperbolic eccentric anomaly	Calculated R AU	Error
1983 Jun 13	30.080	Neptune	3.1095630	1.9988781	30.08000	0.00000
1998 Feb 17	69.419	Reception	8.0949524	2.7360831	69.41908	0.00008
2002 Mar 02	79.830	Reception	9.4651676	2.8647382	79.82822	-0.00178
2002 Apr 27	80.220	Last reception	9.5172247	2.8692893	80.22171	0.00171

Table I lists four observations that we can use to determine these elements by minimizing the least-squared error. Pioneer 10 achieved a hyperbolic orbit after its close encounter with Jupiter on December 3,

1973. We treat subsequent observations as if the spacecraft had been a comet, and we compute its equivalent time and distance of perihelion and its velocity at perihelion. Szebehely[2][2] gives the radial distance from the Sun as

$$r = a\left(e\cosh F - 1\right), \qquad (1)$$

where *a* is the semi-major axis and *e* is the eccentricity of the orbit and *F* is the hyperbolic eccentric anomaly. At the perihelion, $r = r_p$ and $F = 0$. Since cosh(0) = 1, we can solve (1) for *e*.

$$e = 1 + \frac{r_p}{a} \qquad (2)$$

The total energy per unit mass *k* for a hyperbolic orbit is positive (negative for an elliptical orbit). Szebehely[3][3] prefers to define the semi-major axis *a* as a positive number for hyperbolic orbits, so he defines

$$a = \frac{\mu}{k} \qquad (3)$$

where $\mu = GM_{sun} = 1.3271244\cdot10^{20}\ m^3/s^2$ is the heliocentric gravitational constant.

From this we can compute the mean motion *n* in radians per second[4][4] if we express *a* in meters.

$$n = \sqrt{\frac{\mu}{a^3}} \qquad (4)$$

The mean anomaly *M* at any time *T* is given by

$$M = n\left(T - T_p\right) \qquad (5)$$

where T_p is the time of the perihelion. The hyperbolic eccentric anomaly is obtained by solving Kepler's equation for hyperbolic orbits for *F*.

$$M = e\left(\sinh F\right) - F \qquad (6)$$

I let k, r_p, and T_p be the independent variables, and I used Microsoft Excel Solver to minimize the mean squared error for the four observations listed in Table I. The results are as follows.

$k = 126{,}802{,}006\ m^2/s^2$
$r_p = 2.869424583$ AU
$T_p = $ 1974 Apr 15 at 21:47:36 UT
$a = 6.9961658669$ AU
$e = 1.410142446$
$n = 9.29589684\text{E-}04$ radian/day

So, for a given time *T*, compute *M* from Eq. (5), then compute *F* from Eq. (6), and compute *r* from Eq. (1).

3. Newtonian Acceleration

The Newtonian acceleration towards the Sun is simply the force per unit mass, which is the inverse square law. Since the symbol *a* is used for the semi-major axis, we will use *dv*/*dt* for acceleration.

$$\frac{dv}{dt} = \frac{\mu}{r^2} \qquad (7)$$

The result is plotted in Figure 1.

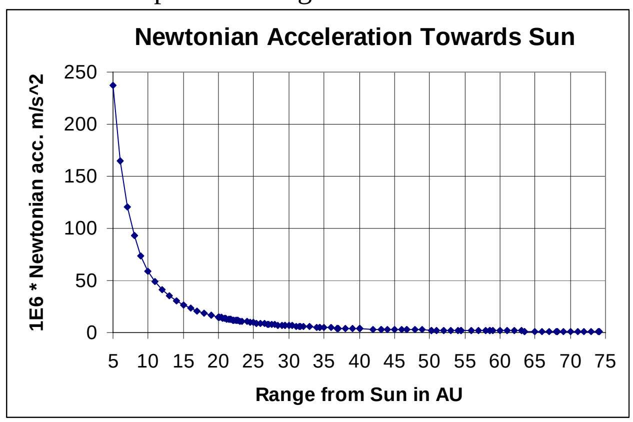

Figure 1. Newtonian Acceleration

The anomalous acceleration cannot be seen at this scale because it is about 10,000 times smaller.

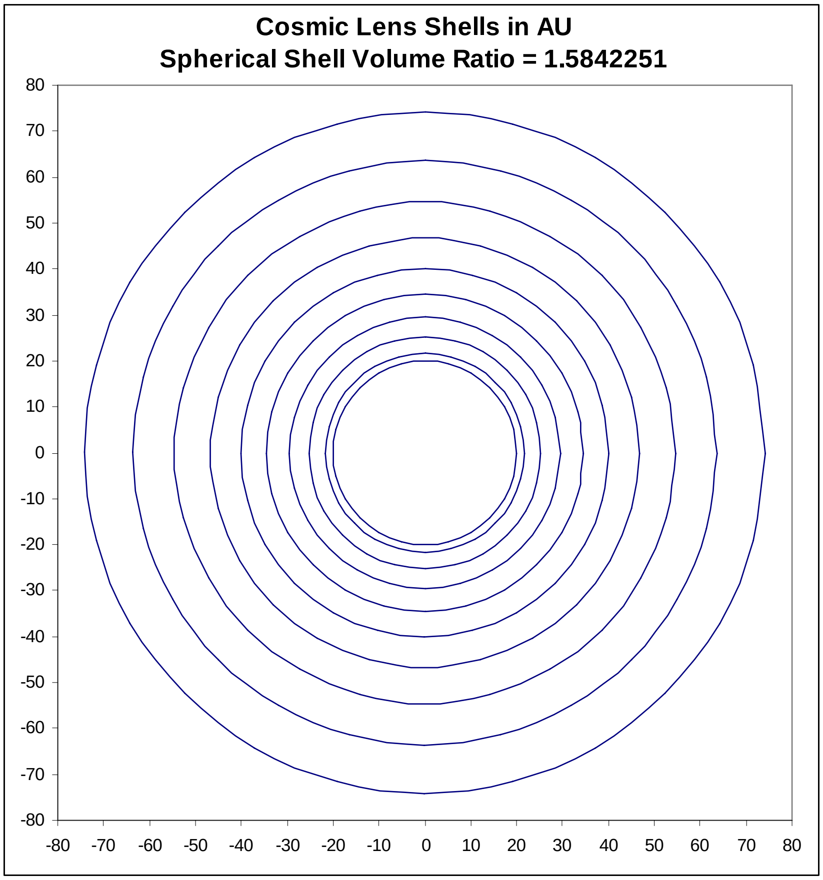

Figure 2. Cross-Section of Cosmic Lens Shells in AU

4. Cosmic Lens Structure

Figure 2 is a cross-section of nine concentric spherical shells that comprise the cosmic lens model that is the subject of this paper. The inside radius of the innermost layer is 20 AU which corresponds to the onset of the Pioneer 10 anomaly. To get an idea of the scale of this lens, recall that the average radius of the orbit of Uranus is 19.1829 AU. The effective refraction index, n, at any point inside any one of the layers is the ratio of the speed of light in that layer to the speed of light inside the 20-AU boundary, which is what we measure in the laboratory. The progression of the refraction index was determined by the constraint of the error bars on the observed anomalous acceleration of $(8.74 \pm 1.33) \cdot 10^{-10}$ m/s^2. This progression is illustrated in Figure 3. Figure 3 plots $n - 1$ to exaggerate the stair-step pattern.

If there were only eight layers, the outer radius

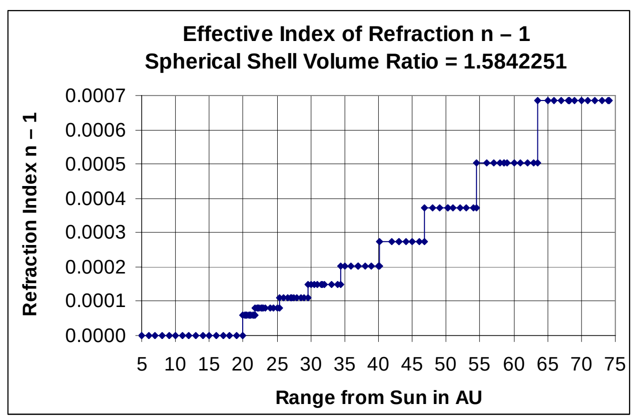

Figure 3. Effective Refraction Index *n* – 1

of the lens would be 63.55 AU, and the effective refraction index outside the lens (to infinity) would be 1.0006858. That would be the ratio of the speed of light outside the lens to the speed of light inside the 20-AU boundary.

5. Cause of the Anomaly

The effect of this stair-step in refraction index (speed of light ratio) is to induce an apparent anomalous sunward acceleration to the spacecraft according to Doppler observations. That is because what Doppler observations observe is the ratio v/c. Observers see an anomalous reduction in that ratio, and they assume that means a reduction in v, since the conventional wisdom, according to Einstein's postulate, is that the speed of light, c, is everywhere constant. But this model asserts that c increases as the spacecraft passes through each ether discontinuity at each lens layer boundary. Therefore even if v remained constant as the spacecraft passed though a discontinuity in the ether, the increase in c on the other side of the discontinuity would cause a reduction in the ratio v/c that causes the Doppler frequency shift. Figure 4 illustrates the apparent anomalous acceleration in excess of Newtonian acceleration.

The nominal anomalous acceleration of $8.74 \cdot 10^{-10}$ m/s^2 is plotted as the red horizontal line.

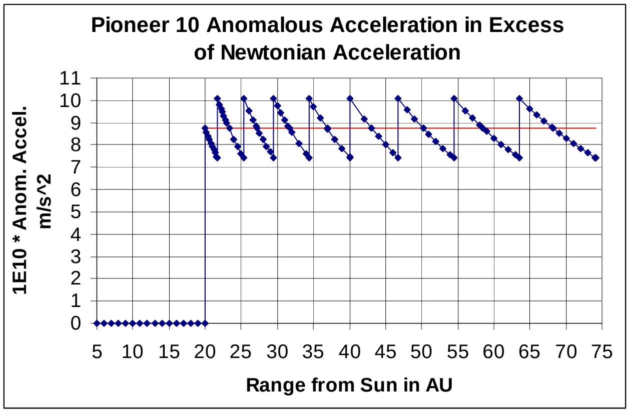

Figure 4. Anomalous Acceleration

The saw-tooth limits correspond to the excursions of $\pm 1.33 \cdot 10^{-10}$ m/s^2. Each of the slanting curves corresponds to the falling off by the inverse square of the distance. If there were only eight layers, the last curve on the right would just continue curving downward towards zero acceleration as the range approached infinity. The results are summarized in Table II.

Table II. Cosmic Lens Spherical Shell Parameters

Shell	inner R AU	outer R AU	Refract. Index n	Shell Volume m^3	Volume Ratio
1	20.000000	21.720840	1.000058954	3.152237E+37	
2	21.720840	25.321081	1.000080116	8.396048E+37	2.6635209
3	25.321081	29.518064	1.000108876	1.330123E+38	1.5842251
4	29.518064	34.410700	1.000147960	2.107214E+38	1.5842251
5	34.410700	40.114292	1.000201074	3.338302E+38	1.5842251
6	40.114292	46.763259	1.000273254	5.288621E+38	1.5842251
7	46.763259	54.514295	1.000371346	8.378367E+38	1.5842251
8	54.514295	63.550070	1.000504649	1.327322E+39	1.5842251
9	63.550070	74.083529	1.000685805	2.102777E+39	1.5842251

6. Computing the Anomaly

The "observed" acceleration is equal to the product of the refraction index times the Newtonian acceleration computed from Eq. (7), and the anomalous acceleration is equal to their difference. So the anomalous acceleration is equal to $(n - 1)$ times the Newtonian acceleration. At 20 AU the refraction index was adjusted to give an anomalous acceleration of $8.74 \cdot 10^{-10}$ m/s^2. This refraction index of 1.000058954 was held constant and r was increased until the anomaly had reached the lower limit of $(8.74 - 1.33) \cdot 10^{-10}$ m/s^2 = $7.41 \cdot 10^{-10}$ m/s^2. At that point, r = 21.720840 AU. Then with r held constant at that value, the refraction index was increased until the anomaly had reached the upper limit of $(8.74 + 1.33) \cdot 10^{-10}$ m/s^2 = $10.07 \cdot 10^{-10}$ m/s^2. At that point the refraction index was held constant at this value of 1.000080116 and r was increased until the anomaly had reached the lower limit of $7.41 \cdot 10^{-10}$ m/s^2. At that point, r = 25.321081 AU. Then with r held constant at that value, the refraction index was increased until the anomaly had reached the upper limit of $10.07 \cdot 10^{-10}$ m/s^2. This process was repeated until the 9th shell was filled out. See Fig. 5.

7. Shell Volume Ratio

An interesting but unexpected result of this procedure is that the ratio of adjacent individual shell volumes is a constant having a value of 1.5842251 as shown in the last column of Table II. The value of this constant depends only on the error bars $\pm 1.33 \cdot 10^{-10}$ m/s^2 for the anomaly. The first ratio is different because the first shell is not yet filled.

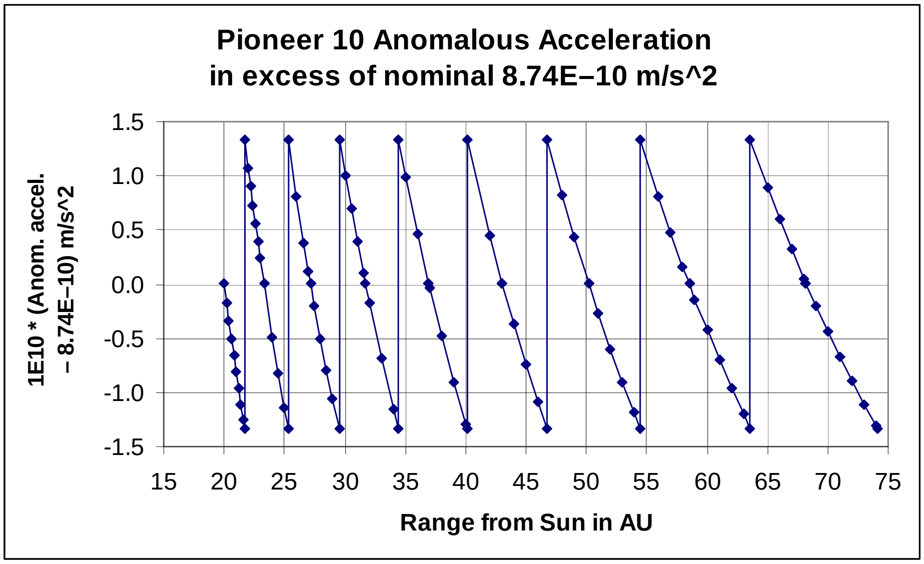

Figure 5. Pioneer 10 Anomalous Acceleration in Excess of Nominal $8.74 \cdot 10^{-10}$ m/s^2

References

[1] [1] John D. Anderson, *et al*., "Study of the anomalous acceleration of Pioneer 10 and 11", Phys. Rev. D **65**, 082004(2002), Issue 8—April 2002.

[2] [2] Victor G. Szebehely, *Adventures in Celestial Mechanics*, The University of Texas Press, 1989, p. 77.

[3] [3] *ibid*, p. 73.

[4] [4] *ibid*, p. 82.